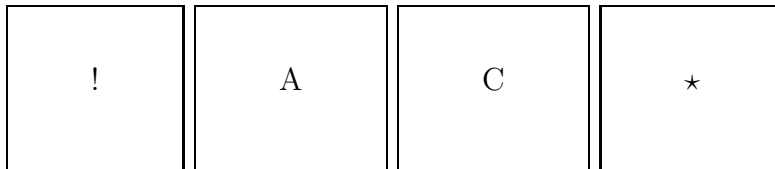


Exam I Solutions, MA1025, Winter 2004

1. Suppose we are shown four cards that are lying on a table. We know that one side of each card is labeled either A, B, or C, and that the other side of each card is labeled either !, \star , or Δ . The cards appear as follows:



We are told that if any card bears an B or a C on one side then it must bear a \star or a Δ on the other side. Precisely which cards need to be turned over to either verify or refute this claim? Explain.

Solution: The card showing a ! must be turned. If it has either a B or a C on the reverse, the claim is false. The cards bearing A and \star are of no interest. The card bearing a C must be turned; if the other side shows neither \star nor Δ , the claim is false. If neither of the two conditions described as falsifying the claim occur, the claim is true.

2. Assume that the universal set is the set \mathbf{R} . Decide whether each of the following is true or false, and in each case briefly explain your reasoning.

(a) $\forall y \exists! x (x = |y|)$ is true, since the absolute value function is well defined for all reals.

(b) $\exists! x \forall y (x = |y|)$ is false, since it asserts that there is some “magic” x that is simultaneously the absolute value of every real number. This indirectly asserts that either there are only two real numbers or that there is only one.

(c) $\forall x \exists! y (x = |y|)$ is false. If $x < 0$, then $x = |y|$ is false for all reals, while if $x > 0$ then $x = |y|$ is true for two reals, namely y and $-y$.

(d) $\forall x (x \geq 0 \Rightarrow \exists y (x = |y|))$ is true. If $x \geq 0$, then $x = |x|$ and $x = |-x|$.

3. This problem concerns set notation.

(a) Describe each set using the form $\{f(x) | P(x)\}$:

i. $A = \{1, 3, 9, 27, \dots\} = \{3^{n-1} | n \in \mathbf{N}\}$

ii. $B = \{1, 1/4, 1/9, 1/16, \dots\} = \{1/n^2 | n \in \mathbf{n}\}$

(b) Describe each set as a list of elements between braces:

i. $\{x \in \mathbf{Z} \mid -8 \leq x^3 \leq 8\} = \{-2, -1, 0, 1, 2\}$

ii. $\{x \in \mathbf{N} \mid (\exists y \in \mathbf{N})(x^2 + y^2 \leq 20)\} = \{1, 2, 3, 4\}$

4. Let $A = \{1, 2, 3\}$, $B = \{1, \{2, 3\}\}$, and $C = \{1, 2, 3, \{1, 2, 3\}\}$. Find the following:

(a) $A \cup C = C$

(b) $A \cap B = \{1\}$

(c) $B \cup C = \{1, 2, 3, \{2, 3\}, \{1, 2, 3\}\}$

(d) $B \cap C = \{1\}$

(e) $B - C = \{\{2, 3\}\}$

(f) $A - B = \{2, 3\}$